HB 5 College Preparatory Math Content Framework

Target Students: This course is appropriate for 12th grade students whose performance on measures outlined in TEC §28.014 indicates that the student is not ready to perform entry-level college coursework in Mathematics. This course is designed to advance college and career readiness.

Recommended Prerequisites: Satisfactory completion of Algebra I and the Algebra I EOC exam, Geometry, and a third credit of mathematics from the courses listed in TAC §74.12(b)(1)(2).

Course Description:
This course addresses a variety of mathematical topics needed to prepare students success in college-level mathematics. In this course students will connect and use multiple strands of mathematics in situations and problems, as well as in the study of other disciplines. In addition, the course supports students in developing skills and strategies needed to succeed in college. Mathematics topics include: numeracy with an emphasis on estimation and fluency with large numbers; manipulating and evaluating expressions and formulas, to include perimeter, area, and volume; rates, ratios, and proportions; percentages; solving equations; linear equations and inequalities; linear systems; exponential models; data interpretations including graphs and tables; verbal, algebraic, and graphical interpretations of functions. Mathematical process standards are also included in this framework; these process standards describe ways in which students are expected to engage in the content. Successful completion of this course, as defined by the memorandum of understanding (MOU) with the partnering institution(s), grants the student an exemption to TSI requirements for mathematics at the partnering institution(s).

Course Goal:
The goal of this course is to develop students' quantitative and algebraic reasoning abilities, thus preparing them for college success in multiple mathematics pathways.

Student Learning Outcomes & Learning Objectives:
Upon successful completion of this course, students will be able to:

(1) Mathematical process standards. Use mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

- (A) apply mathematics to problems arising in everyday life, society, and the workplace;
- (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
- (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
- (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
- (E) create and use representations to organize, record, and communicate mathematical ideas;
- (F) analyze mathematical relationships to connect and communicate mathematical ideas; and
- (G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

(2) Numeric reasoning. Solve authentic problems in a variety of contexts that require number sense and the ability to apply concepts of numeracy to investigate and describe quantitative relationships. Students will be able to:

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(A) Engage in problem-solving which demonstrates operation sense and communicates verbally and symbolically with real and complex numbers.

For example: Predict the effects of multiplying a number by a number between 0 and 1. Represent real numbers on a number line and complex numbers on the complex plane. Use the order of operations to simplify expressions, to solve problems, and to identify errors in a spreadsheet. Perform operations on real and complex numbers and connect operations on real numbers to operations on complex numbers.

(B) Engage in problem-solving which demonstrates an understanding of rational numbers in the form of fractions, decimals, and percentages by representing quantities in equivalent forms, comparing the size of numbers in different forms and interpreting the meaning of numbers in different forms.

For example: Write a fraction in equivalent decimal form and vice versa. Compare growth expressed as a fraction versus as a percentage. Interpret the meaning of fractions and percentages. Interpret the meaning of percentages greater than 100% and justify whether such a percentage is possible in a given context.

(C) Solve problems involving calculations with percentages and interpret the results.

For example: Calculate a percentage rate. Explain the difference between a discount of 30% and two consecutive discounts of 15%. Calculate relative change, represent it graphically, and explain how it differs from absolute change.

(D) Demonstrate an understanding of large and small numbers by interpreting and communicating with different forms (including words, fractions, decimals, standard notation, and scientific notation) and compare magnitudes.

For example: Compare large numbers in context, such as the population of the US compared to the population of the world. Calculate ratios with large numbers such as water use per capita for a large population. Interpret a growth rate less than 1%.

(E) Use estimation skills, and know why, how, and when to estimate results.

For example: Identify and use numeric benchmarks for estimating calculations. Identify and use contextual benchmarks for comparison to other numbers. Check for reasonableness using both types of benchmarks.

(F) Solve problems involving measurement including the correct use of units.

For example: Identify the appropriate units for perimeter, area, and volume. Calculate the amount of paint needed to paint a non-rectangular surface.

(G) Use dimensional analysis to convert between units of measurements and to solve problems involving multiple units of measurement and know why and how the process works and when to use it.

For example: Convert between currencies. Calculate the cost of gasoline to drive a given car a given distance. Calculate dosages of medicine.

(H) Read, interpret, and make decisions about data summarized numerically (e.g., measures of central tendency and spread), in tables, and in graphical displays (e.g., line graphs, bar graphs, scatterplots, and histograms).

For example: Critique a graphical display by recognizing that the choice of scale can distort information. Explain the difference between bar graphs and histograms. Explain why the mean may not represent a typical salary.

(3) Proportional reasoning. Represent and solve authentic problem situations using proportional reasoning with ratios, rates, proportions, and scaling. Students will be able to:

(A) Model real-world situations using ratios in a variety of forms (including percentages, fractions, and decimals).

For example: Interpret a rate of change within a context using appropriate units. Interpret a percentage as a number out of 1,000. Compare risks expressed in ratios with unequal denominators (e.g., 1 in 8 people will have side effects versus 2 in 14).

(B) Determine whether a proportional relationship exists based on how the change in one value influences the change in the other, using various representations.

For example: Simple versus compound interest. Analyze whether an estimated percentage is reasonable based on proportions. Analyze the effects of scaling and shrinking that are proportional and non-proportional (e.g., the impact of changing various dimensions on perimeter, area, and volume).

(C) Analyze, represent, and solve real-world problems involving proportional relationships, including indirect measurement, with attention to appropriate use of units.

For example: Use individual water-use rates to predict the water used by a population. Use the Consumer Price Index to compare prices over time. Use a scale to calculate measurements in a graphic.

(4) Algebraic reasoning. Investigate problems that facilitate the transition from specific and numeric reasoning to general and abstract reasoning. Use the language, symbols, and structure of algebra to investigate, represent, and solve those problems. Students will be able to:

(A) Demonstrate understanding of the meaning and uses of variables as unknown quantities in real-life situations, in equations, in simplifying expressions, and as quantities that vary, and use that understanding to represent quantitative situations symbolically.

For example: Understand the different uses of variables and the difference between a variable and a constant. Be able to use variables in context and use variables as placeholders, as in formulas. Write an algebraic expression to represent a quantity in a problem. Use algebraic properties, concepts, procedures (including factoring), and algorithms to combine and transform absolute value, polynomial (2nd, 3rd, and 4th degree polynomial expressions), radical, and rational expressions. Determine whether different expressions are equivalent. Use notation with variables (e.g., exponents, subscripts) in simple and moderately complex expressions.

(B) Analyze real-world problem situations, and use variables to construct and solve equations involving one or more unknown or variable quantities. Be able to use a variety of representations to justify whether different equations are equivalent.

For example: Construct and solve linear, absolute value, and polynomial (2nd and 3rd degree polynomial functions), radical, and rational equations. Demonstrate understanding of the meaning of a solution. Write a spreadsheet formula to calculate prices based on percentage mark-up. Solve a formula for a given value. Identify when there is insufficient information given to solve a problem.

(C) Express and interpret relationships using inequality symbols; solve algebraic inequalities.

For example: Use inequalities to express the relationship between the probabilities of two events or the size of two groups. Interpret a histogram based on intervals expressed with inequality symbols. Write and solve inequalities that represent constraints in contextual situation, including linear and absolute value inequalities; demonstrate an understanding of the difference between the solution to an equation and the solution to an inequality.

(D) Determine and justify whether a situation represents multiplicative or additive change, or neither. Use various representations, such as verbal descriptions, tables, and graphs, to compare and contrast the effect of multiplicative or additive change.

For example: Compare and contrast the rate of change and/or behavior of a linear and an exponential relationship in context. Recognize that a multiplicative change is different from an additive change. Explain how the rate of change of a linear relationship differs from an exponential rate of change, as well as the ramifications of exponential change (growth can be very slow for a time but then increase rapidly).

(E) Recognize, understand and analyze features of a linear, exponential, quadratic, rational, or radical function. Justify whether a problem situation represents a linear, exponential, quadratic, rational, or radical relationship (or none). Represent such models using a variety of representations.

For example: Given a set of data, make an informal, intuitive evaluation of the applicability of a particular mathematical model. Given an initial value and information about change, create a table, graph, and/or algebraic model. Given an algebraic model, create a table of values. Recognize the limitations of the model and identify an appropriate domain or range for which the model might be used to make accurate predictions. (Note: Students are not required to use the formal vocabulary of domain and range.)

(F) Construct and use linear and piecewise linear mathematical models to solve problems from a variety of contexts and to make predictions/decisions.

For example: Describe the rate of change (slope) using appropriate units. Determine the contextual meaning of the slope and of the x- and y-intercepts. Use the understanding of linear functions to analyze and construct piece-wise linear functions, including absolute value functions.

(G) Construct and use exponential mathematical models to solve problems from a variety of contexts and to make predictions/decisions.

For example: Describe the rate of change using appropriate units (average rate of change over an interval). Determine the contextual meaning of the x- and y-intercepts, if any. Examples include: Given a statement of

how the balance in a savings account grows at different interest rates, construct a table of months and balances and then write a mathematical model that provides the balance for a given month. Compare simple and compound interest.

(H) Construct and use quadratic models to solve problems from a variety of contexts and to make predictions/decisions.

For example: Solve quadratic equations using a variety of methods, including the zero product property (factoring) and the quadratic formula. Use contextual applications such as the Pythagorean Theorem and projectile motion to set up and solve quadratic equations.

(I) Construct and use rational models to solve problems from a variety of contexts and to make predictions/decisions.

For example: Use algebraic properties (such as factoring) to simplify rational expressions. Use contextual applications such as indirect variation to set up and solve rational equations. Analyze situations modeled by rational functions to solve real world problems.

(J) Construct and use square root models to solve problems from a variety of contexts and to make predictions/decisions.

For example: Use algebraic properties to simplify radical expressions and solve contextual and mathematical radical equations. (Restrict equations to those of the form $\sqrt{ax + b} = c$.) Analyze situations modeled by square root functions to solve real world problems.

(5) Probabilistic reasoning. Use counting principles and probability to quantify uncertainty in a variety of real-world contexts; students will understand and critically evaluate statements that appear in the popular media (especially in presenting medical information) involving risk and arguments based on probability. Students will be able to:

(A) Build a finite sample space to model the outcomes of real-world events by determining the nature and number of elements using counting techniques.

For example: Make lists, tables, and tree diagrams to represent all possible outcomes in determining specifics of the sample space. Determine the number of ways an event may occur using the Fundamental Counting Principle.

(B) Compute and interpret the probability of a real-world event; and compute the probability of its complement and interpret its meaning.

For example: Conduct an experiment or simulation to compute the empirical probability of an event and its complement. Compute and interpret the theoretical probability of a simple event and its complement.

(C) Compute and interpret the probability of conditional and compound events.

For example: Determine whether real-world events are independent or dependent. Explain the meaning of conditional probability and know when to use it. Compute conditional and joint probabilities from a given table of data.

(D) Interpret statements about chance, risk, and probability that appear in everyday media (including terms like unlikely, rare, impossible).

For example: Interpret statements such as "for a certain population the risk of a particular disease is 0.005". Compare incidences of side effects in unequal group sizes.

(E) Identify common pitfalls in reasoning about risk and probability.

For example: Identify inappropriate risk statements, such as when the size of reference groups is unknown (e.g., California, 2009, 88% of motorcycle accident fatalities were helmeted, 12% unhelmeted).

(F) Interpret in context marginal, joint, and conditional relative frequencies in context for data summarized in a two-way table and identify which relative frequency is appropriate to answer a contextual question.

For example: Distinguish between reported relative frequencies that are marginal, joint, or conditional. Choose the relative frequency that is the most informative for a given purpose. Choose the appropriate direction of conditioning for a given context (the chance of cancer given a positive test result is not the same as the chance of a positive test result given cancer).

(G) Demonstrate understanding of absolute risk and relative risk (percentage change in risk) by describing how each provides different information about risk.

For example: Interpret the different information conveyed when comparing the magnitude of the absolute risks and percentage change in risk (e.g., an 80% increase in risk associated with taking a particular medication could mean a change in risk from 0.001 to 0.0018 or from 0.1 to 0.18).

(6) Quantitative Reasoning in Everyday Life. Understand, interpret, and make decisions based on financial information commonly presented to consumers; Students will understand that quantitative information presented in the media and by other entities can sometimes be useful and sometimes be misleading. Students will be able to:

(A) Demonstrate understanding of common types of consumer debt and explain how different factors affect the amount that the consumer pays.

For example: Calculate the interest paid on credit card debt based on a credit score; explain how the length of the pay-off period affects the total interest paid; demonstrate the relationship between a percentage rate and the amount of interest paid; define basic terminology such as principal, interest rate, balance, minimum payment, etc.

(B) Demonstrate understanding of compound interest and how it relates to saving money.

For example: Demonstrate the different impacts of the saving period and the amount saved on the accumulated balance; use a given formula to calculate a balance; demonstrate an understanding of the meaning of a compounding period and use the appropriate terminology for different periods (e.g., quarterly, annually, etc.).

(C) Identify erroneous or misleading information in advertising or consumer information.

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For example: Explain why statements about "average" benefits of a product such as a weight loss plan are misleading; identify misleading graphs that create an appearance of greater impact than is warranted.

(D) Use quantitative information to explore the impact of policies or behaviors on a population. This might include issues with social, economic, or environmental impacts.

For example: Calculate the effects of a small decrease in individual water use on the amount of water needed by a large population over time; determine if the minimum wage has kept pace with inflation over time.

(E) Identify erroneous, misleading, or conflicting information presented by individuals or groups regarding social, economic, or environmental issues.

For example: Explain how two statements can be both contradictory and true (e.g., the "average" amount of a tax cut expressed in terms of the mean and the median); identify how two pie charts representing different populations can be misleading.

This framework for the HB 5 College Preparatory Course in mathematics is the result of a collaborative process organized by the Texas Success Center (TSC) at the Texas Association of Community Colleges (TACC). A committee consisting of content experts from both Higher Education and K-12 educators worked collaboratively to develop and revise this framework, seeking input from educators across the state. This feedback, along with the Learning Outcomes in the Lower-Division Academic Course Guide Manual (ACGM), the Texas College and Career Readiness Standards (CCRS), and other relevant materials informed the committee's work. The HB 5 College Preparatory Course framework development is an ongoing, rigorous process designed to be inclusive of voices from multiple stakeholders with the goal of producing exceptional course materials. If you wish to contribute feedback that would benefit the continued development of this framework, please e-mail HB5Math@tacc.org.