Lesson 15, Parts C and D
Do You Trust the Test?

Preparing for the lesson

Overview	This lesson provides students with practice analyzing two-way tables while exploring false positives and false negatives in test results. Consider using the attached Alternative Opener to introduce this lesson.
Objectives	Students will understand that: • A percent can be used to express the likelihood (or probability) of a certain event. • Selecting the appropriate comparison value and base value is crucial for calculating a percent correctly. Evaluating the accuracy of a test can depend on what data on which you focus. Students will be able to: • Extract relevant information from a two-way table. • Select the appropriate values to calculate probabilities. • Calculate and interpret probabilities of errors: percent of positive results that are false positives and percent negative results that are false negatives.
Materials and technology	• Computer, projector, document camera • Preview Assignment 15.CD • Student Pages for Lesson 15, Part C • Practice Assignment 15.CD
Prerequisite assumptions	Before beginning this lesson, students should be able to: • Read a two-way table with understanding. • Have a basic understanding of the meaning of percent and the notation used to describe percent (%). • Use a calculator to divide two numbers and interpret the resulting decimal representation as a percent. • Calculate percentages.
Making connections	This lesson: • Connects back to Lesson 14, Part A. • Connects back to the previous work with two-way tables, and

	the previous work with probability, including Resource **Probability, Chance, Likelihood, and Odds**.
	• Connects forward to topics in statistics like Chi-square tests, Bayes' Theorem, and Simpson's Paradox.
Productive struggle level	2

In Lesson 15.C, students will encounter four important percentages or probabilities:

1) *Sensitivity*, the probability that a medical test indicates that a disease is present when a patient has the disease (the ratio of true positives to affected patients).

2) *Specificity*, the probability that a medical test indicates no disease when a patient does not have the disease (the ratio of true negatives to unaffected patients).

3) The probability that a negative result is wrong (the ratio of false negatives to negative results).

4) The probability that a positive result is wrong (the ratio of false positives to positive results).

The first two probabilities relate to the accuracy of the test, while the last two quantify inaccuracies in the test. All four of these probabilities are different and independent of one another. That is, knowing one of the above does not allow you to compute any other easily.

Suggested instructional plan

Part C (25 minutes)

Frame
(4 minutes)

Note: If you use the Alternative Opener to this lesson, complete that activity and then transition by saying, "We are going to look at a larger sample of strep throat data."

- Display or distribute the Student Pages for this lesson.
- Have students read the top half of the first page and answer the prompt.
- Transition to the lesson activities by briefly discussing the **Objectives for the lesson**.

Lesson activities
(18 minutes)

Question 1

- This type of question was covered in Lesson 15, Part A. Students will probably not have much trouble with it.
- Ask individual students to volunteer explanations for each number. For example, "19 patients tested positive for strep throat despite not having a Streptococcal infection."

Questions 2 and 3

Check for Understanding

- Students may have difficulty identifying which numbers to use.
- Point to various cells in the table and ask, "What does this number represent?"
- "Tell me what information you need (in words) for this calculation. Where is that information in the table?"

Literacy Support

- The lesson introduces the terms *sensitivity* and *specificity*, but the language is not the focus of the lesson.
  - The *sensitivity* of a strep throat test equals the ratio of true positive results to the total number of affected patients (those with strep throat).
  - The *specificity* of a strep throat test equals the ratio of true negative results to the total number of unaffected patients (those who do not have strep throat).

Class

- Discuss the answers and the calculations.

Discussion

- o "What sort of errors did you make in calculating the percentages?" (You might want to refer to a specific problem if you observed students using the wrong comparison or base value.)
- o "What was most important (or most helpful) in deciding which numbers to use while calculating the percentages?"
- o "What do you notice about the data?"
- The next lesson formally introduces false positives and false negatives, but students may notice the incidence of inaccurate results now.

Extension Question

- "Assume the sample in the lesson represents the general population, that is, the data are proportional to the overall number of patients complaining of a sore throat. If there are 6,300 sore-throat patients, how many probably have strep throat?" [Answer: 1,512]

Wrap-up/transition

(3 minutes)

Wrap-up

- Note the importance of identifying the correct comparison value and base value in calculating percentages. This connects to Lesson 15, Part A.
- Tell students that there will be a more in-depth evaluation of the test results in the next lesson.
- Have students refer back to the **Objectives for the lesson** and check the ones they recognize from the activity. Alternatively, they may check objectives throughout the lesson.

Transition

- Explain that students will be learning about the danger of false test results.

Part D (25 minutes)

Frame

(3 minutes)

- Ask students "Have you ever been misdiagnosed? Has your doctor ever treated you for an illness, but you did not respond to the treatment? Did you wonder how much guesswork goes into medical diagnosis?"
- Briefly discuss the issue of errors in tests. You might cite examples related to this context or other medical tests, including drug-use screenings.
- Transition to the lesson activities by briefly discussing the **Objectives for the lesson**.

Lesson activities

(19 minutes)

- Have students read the lesson opener before starting questions 1–4.

Questions 1–4

Group Work

Guiding Questions

- Students may have difficulty identifying which numbers to use.
  - Point to various cells in the table and ask, "What does this number represent?"
  - "What test result is this question talking about? So what is the total number with this result?"
  - "Tell me what information you need (in words) for this calculation? Where is that information in the table?"
  - "Think about the ratio of incorrect negative results compared to all negative results."
  - "Of patients with this test result, what group is being compared to the total in that row?"

Whole Class Debrief

- You may wish to debrief questions 1 and 2 to gauge understanding of *false negatives* before directing students to proceed to *false positives* in questions 3 and 4. If so, see the debriefing suggestions in the next section.

Questions 5–6

Literacy Support

- Ask students to consider the difference between this definition of the word *accurate* and the usual definition of accurate. Similarly, the usual meanings of the words *positive*, *negative*, and *false* also may distract or mislead students.

- Remind students that "positive" does not mean "correct." Likewise, "negative" is not synonymous with "incorrect."

- These questions are open-ended enough to allow diverse answers. Some students may use the total number of patients as the base value. For instance, students may divide the number of incorrect results (43) by the total number of patients (500), computing 8.6% as a measure of inaccuracy (or accuracy).

- Allow diverse student response to drive discussion toward the objective, "Evaluating the accuracy of a test can depend on what data you focus on."

- Students who use the number of patients with strep throat as the comparison value and the total number of patients as the base value are not addressing the issue of accuracy/inaccuracy, but they are approaching the issue of *prevalence*, the ratio of diseased patients in a population. *Prevalence* is a lurking variable confounding the accuracy of medical tests.

- Students may have difficulty getting started with these open-ended

questions. Facilitate by "working backward." Have them look at one of their results, such as the answer to question 1.

- o "Imagine you are the doctor. If you are looking at a positive result, how sure are you that the patient has strep throat?"

Whole Class Discussion of Lesson 15, Parts C and D

This discussion should include all the work from the previous lesson. You might want to write the summary results of the questions from the previous lesson as well as questions 1 and 2 on the board so that students can see all of them together.

- Discuss the answers and the calculations in questions 1 and 2.
  - o "What sort of errors did you make in calculating the percentages?" (You might want to refer to a specific problem if you observed students using the wrong comparison or base value.)
  - o "What was most important (or most helpful) in deciding which numbers to use in calculating percentages?"
  - o "Did any of the results surprise you?"
- Discuss the responses to questions 3 and 4.
  - o "What do you think about the overall accuracy of the test?"
  - o "What are the implications of a false-positive drug test?" [Answer: Patients suffer delayed recovery from illness. Patients suffer added expense and possibly complications from the unnecessary treatment.] Point out that this is why doctors must weigh the patient's symptoms against the test result.
- As you have time, you can discuss the risks involved with misdiagnosis:
  - o Undiagnosed strep throat can lead to delayed recovery.
  - o Overtreatment of strep throat with antibiotics may lead to drug-resistant strains of the bacterium.

Wrap-up/transition

(3 minutes)

Wrap-up

- As with other lessons, it is important to discuss that one goal of quantitative literacy is to help people make informed decisions based on evidence. This lesson illustrates how important it is to look at data from many angles. The lesson does not mean "all tests are bad," but neither should you put unquestioning faith in a medical test.

- "What are the implications of a false-negative strep throat test?" [Answer: A false-positive result can mean that another disease goes untreated. Kawasaki Disease mimics many strep throat symptoms. If left untreated, Kawasaki Disease can cause severe or even fatal heart complications (see the Kawasaki Disease Foundation at www.kdfoundation.org). The television show *Grey's Anatomy* highlighted the case of a false-positive strep test that

actually delayed the treatment of a small boy with Kawasaki Disease (Season 9, Episode 19, original airdate, March 28, 2013).]

- Have students refer back to the **Objectives for the lesson** and check the ones they recognize from the activity. Alternatively, they may check objectives throughout the lesson.

Transition
- Highly contagious diseases like strep throat spread more readily in dense populations. The next lesson investigates population density.

Suggested assessment, assignments, and reflections

- Give Practice Assignment 15.CD.

- Give the Preview Assignments, if any, for the lesson activities you plan to complete in the next class meeting.

Alternative Opener

Overview

Using a context such as the reliability of strep throat testing is very relevant to students, and false results can have large consequences. The debrief for Lesson 15, Part D can be done even if you choose not to do this opening activity. It points out the danger of false results: A false-positive result can mean that another disease goes untreated. For example, Kawasaki Disease mimics many strep throat symptoms and, if left untreated, can cause severe or even fatal heart complications. (See the Kawasaki Disease Foundation at www.kdfoundation.org.)

Suggested resources and preparation

Materials and technology

- For a class with 25 students, prepare a paper bag with the following strips of paper:
  - 1 strip says: "You have a very sore throat and visit the clinic. They test you for strep throat. The test results say you do have strep throat but you really don't."
  - 2 strips say: "You have a very sore throat and visit the clinic. They test you for strep throat. The test results say you don't have strep throat but you really do."
  - 10 strips say: "You have a very sore throat and visit the clinic. They test you for strep throat. The test results say you have strep throat and you really do."
  - 12 strips say: "You have a very sore throat and visit the clinic. They test you for strep throat. The test results say you don't have strep throat and you really don't."
- Put a sign in each corner of the room and have students stand in the appropriate corner.
- These strips and signs are available in this document, so be sure to duplicate these pages single-sided.
- For a different class size, adjust the numbers accordingly, ensuring that you have a very small number of "no strep, but positive test result" and "strep, but negative test result" strips.

Suggested instructional plan

Opener activity
(9 minutes)

- As students enter the room, have them draw a slip from the paper bag. Direct students to the appropriate corner.

- Have students count the number in their group and report to the class.

- Post these numbers on the board.

- Point out to students that the corners of the room are the cells of a two-way table. Ask students to create a two-way table that displays the information collected.

- Point out to students that, in Lessons 4 and 5, we were working with *quantitative data*, numerical descriptions of the subjects/participants. In this set of lessons, we are working with *categorical descriptions*: "sick," "well," "positive result," "negative result." The numbers in the problem are the *counts* of the number of subjects in each category. Two descriptions for each subject can be well organized into two-way tables.

Transition
(1 minute)

- "We are going to look at a larger sample of strep throat data."

False Positive	You have a very sore throat and visit the clinic. They test you for strep throat. The test results say you do have strep throat but you really don't.
False Negatives	You have a very sore throat and visit the clinic. They test you for strep throat. The test results say you don't have strep throat but you really do.
	You have a very sore throat and visit the clinic. They test you for strep throat. The test results say you don't have strep throat but you really do.

True Positives	You have a very sore throat and visit the clinic. They test you for strep throat. The test results say you have strep throat and you really do.

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You have a very sore throat and visit the clinic. They test you for strep throat. The test results say you have strep throat and you really do. |

True Negatives	You have a very sore throat and visit the clinic. They test you for strep throat. The test results say you don't have strep throat and you really don't.

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You don't have strep throat, but the test says that you do.

You do have strep throat, but the test says that you don't.

You
do have
strep throat,
and
the test says
that you do.

You don't have strep throat and, the test says you don't.